11-1 Study Guide and Intervention

Sequences

A **sequence** is an ordered list of numbers. Each number is called a **term**. An **arithmetic sequence** is a sequence in which the difference between any two consecutive terms is the same. This difference is called the **common difference**. To find the next term in the sequence, add the common difference to the last term. *✗ add/subtract to get from term to term.*

EXAMPLE 1 State whether the sequence $-4, -1, 2, 5, 8, \ldots$ is arithmetic. If it is, state the common difference. Write the next three terms of the sequence.

$-4, \quad -1, \quad 2, \quad 5, \quad 8$
   $+3 \quad +3 \quad +3 \quad +3$

$d = 3$

Notice that $-1 - (-4) = 3$, $2 - (-1) = 3$, and so on. The terms have a common difference of 3, so the sequence is arithmetic.

$8 + 3 = 11$, $11 + 3 = 14$, $14 + 3 = 17$. The next three terms are 11, 14, and 17.

A **geometric sequence** is a sequence in which the ratio between any two consecutive terms is the same. This ratio is called the **common ratio**. To find the next term in the sequence, multiply the last term by the common ratio. *✗ multiply/divide to get from term to term.*

EXAMPLE 2 State whether the sequence $-1, 2, -4, 8, -16, \ldots$ is geometric. If it is, state the common ratio. Write the next three terms of the sequence.

$-1, \quad 2, \quad -4, \quad 8 \quad -16$
  $\times(-2) \quad \times(-2) \quad \times(-2) \quad \times(-2)$

$r = -2$

Notice that $\frac{2}{-1} = -2$, $\frac{-4}{2} = -2$, and so on. The terms have a common ratio of -2, so the sequence is geometric.

$-16 \times -2 = 32$, $32 \times -2 = -64$, $-64 \times -2 = 128$.
The next three terms are 32, -64, and 128.

Some sequences are neither arithmetic nor geometric. To extend a sequence like this, look for a pattern in the consecutive differences or consecutive ratios. Then apply the pattern to the last term of the sequence.

EXERCISES

State whether each sequence is *arithmetic*, *geometric*, or *neither*. If it is arithmetic or geometric, state the common difference or common ratio. Write the next three terms of the sequence.

1. $0, 3, 6, 9, 12, \textit{15, 18, 21}$
   $+3 \ +3 \ +3 \ +3$ *arithmetic $d = 3$*

2. $3, 6, 12, 24, 48, \textit{96, 192, 384}$ *geometric $r = 2$*
   $\times 2 \ \times 2 \ \times 2 \ \times 2$

3. $6, 11, 16, 21, 26, \textit{31, 36, 41}$
   $+5 \ +5 \ +5$ *arithmetic $d = 5$*

4. $0, 1, 3, 6, 10, \ldots$ *neither*
   $+1 \ +2 \ +3 \ +4$

5. $\frac{1}{9}, \frac{1}{3}, 1, 3, 9, \ldots$
   $\times 3 \ \times 3 \ \times 3 \ \times 3$ *geometric $r = 3$*

6. $30, 26, 22, 18, 14, \ldots$ ~~geometric~~
   $-4 \ -4 \ -4 \ -4$ *arithmetic $d = -4$*

11-1 Practice: Skills

Sequences

State whether each sequence is *arithmetic*, *geometric*, or *neither*. If it is arithmetic or geometric, state the common difference or common ratio. Write the next three terms of the sequence.

1. 1, 5, 9, 13, 17, . . .

2. −3, −6, −12, −24, −48, . . .

3. 2, 5, 8, 11, 14, . . .

4. 1, 6, 16, 31, 51, . . .

5. 3, 7, 11, 15, 19, . . .

6. 4, 9, 14, 19, 24, . . .

7. 1, −3, 9, −27, 81, . . .

8. 1, 2, 1, 2, 1, . . .

9. 7, 4, 1, −2, −5, . . .

10. −4, 4, −4, 4, −4, . . .

11. −5, 10, −20, 40, −80, . . .

12. 22, 25, 28, 31, 34, . . .

13. 0, −2, −5, −9, −14, . . .

14. 4, 12, 36, 108, 324, . . .

15. −2, 10, −50, 250, −1,250, . . .

16. −80, −76, −72, −68, −64, . . .

17. 448, 224, 112, 56, 28, . . .

18. 2, 2, 4, 12, 48, . . .

19. 3, $6\frac{1}{2}$, 10, $13\frac{1}{2}$, 17, . . .

20. 81, 27, 9, 3, 1, . . .